

## An effective continuous-time formulation for scheduling optimization in a shipbuilding assembly process

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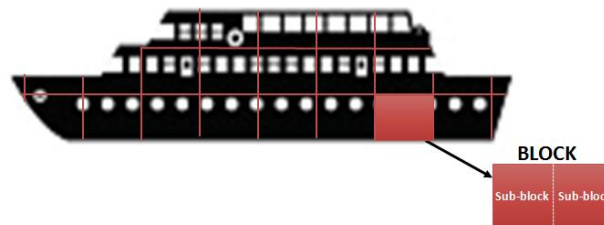
**Abstract.** This work aims at finding an optimal solution of assembly operations in a system of multi-stage production in a shipyard. Shipbuilding of large-size ships is a complex manufacturing process involving the production and assembly of a big quantity of blocks. These blocks are then assembled on the block erection final process, with a predefined order. To achieve competitiveness in this market, the development of efficient operation strategies is a potential alternative. To reach this objective, a mixed-integer linear mathematical model (MILP) is proposed. The model is based on the continuous time-slot time batches concept. This mathematical formulation allows obtaining efficient solutions to academic problems with reasonable computational effort. The MILP problem was tested and computational experiences were reported for industrial problems.

**Keywords:** continuous time-slot, shipbuilding, scheduling, MILP model, shipyard block assembly system

### 1 Introduction

Shipbuilding is a complex manufacturing process which traditionally it carried out by a project-oriented approach. Naturally, each individual ship has some degree of customization and there are only few units based on the same design. Therefore, a modular approach was begun to implement in the last decades taking into account Lean principles and standardizing processes [1]. This approach consists of the use an integrated modular design to construct ships.

Large ships are divided into blocks and they are subsequently assembled in a dry dock. These blocks are the basic units in the shipbuilding process which have different elements incorporated such as pipes, supports, and some electronic equipment. Therefore, the prefabrication of steel blocks or structures is carried out technological advances and more detailed planning. A block consists of the assembly of one or more sub-blocks. The block division of a ship depends on the ship design. This representation of the construction in blocks is shown in Figure 1 which illustrates how two sub-blocks make up a block.



**Fig. 1.** Method of division into blocks - Modular construction

Block-based shipbuilding process involves several stages which require a high degree of coordination between diverse resources. Hence, numerous researches have focused on improving the planning of shipbuilding using different perspectives. For instance, Cho et al. [2] point out that the block assembly process takes more than half of the total shipbuilding processes, so it is very important to have a practically useful block assembly process planning system which can build plans of maximum efficiency requiring minimum man-hours.

Seo et al. [3] and Kim et al. [4] model the problem of the block assembly planning as a constraint satisfaction problem where the precedence relations between operations are considered constraints. To optimize the block spatial scheduling, Shang et al. [5] proposed an allocation algorithm and mathematical model. Cebral et al. [6] and Liu et al. [7] proposed discrete-event simulation based model to achieve an efficient production planning and control.

Many studies have used heuristic algorithms to improve long-term area utilization and minimize processing times of blocks in the planning of the shipbuilding process [8], [9]. On the other hand, methods and algorithms have been proposed recently to solve the scheduling problem in shipbuilding from different approaches, but they do not ensure an optimal solution of the scheduling problem. Nevertheless, a research made by Xiong [10] considered a hybrid assembly-differentiation flowshop scheduling problem and introduced a mixed integer programming (MIP) model to present some properties of the optimal solution. This approach could be useful to the shipbuilding issue, because it could also be considered an assembly flowshop scheduling problem.

In this research, we present the development of a new mixed integer linear mathematical formulation (MILP) to solve the scheduling problem aiming at minimizing the total processing and assembly time of blocks and sub-blocks (makespan) in the yard. The present work aims at finding out the optimal solution of production and assembly operations in a system of multi-stage production of ships of a shipyard while all constraints are satisfied. A ship manufacturing system, which involves a series of production and assembly processes of block and sub-block for large-scale shipbuilding is considered. Hence, a MILP model based on continuous time-slot concept was developed.

This paper is organized as follows. In Section 2, the block assembly process with all stages is described. The model developed with the assumptions and nomenclature

used is presented in Section 3. Next, in Section 4, computational results obtained of the model are shown. Finally, the conclusions are given in Section 5.

## 2 The block assembly process

The shipbuilding process is carried out from the assembly of the hundred blocks of the final structure in the so-called block erection process that is generally performed in a dry dock. Based on the modular approach, the common unit of production for most stages of the process is a block or sub-block. Hence, the manufacturing process of shipbuilding begins with block division. Each block is different in size, type, and consists of one or several sub-blocks assembled, depending on the types of ships. A sub-block is composed of steel plates in accordance with the design drawing of the ship. Both blocks and sub-blocks are considered types of basic intermediate products in the modular design and construction.

In the block assembly process, sub-blocks are assembled in specific workshops to form large blocks. Next, the blocks are assembled in a dock to form the hull of the ship. Therefore, in the early stages of the shipbuilding process steel plates are processed to construct the sub-blocks. In the following stages, the blocks (assembled sub-blocks) are processed and assembled by a given sequence, respecting the specifications of ship assembly.

The main stages of the shipbuilding process are illustrated in Figure 2. The shipbuilding process begins by first stage called *Cutting Steel*, where the welding and cutting processes of steel plates are performed according to the requirements of the sub-blocks designs. Panels, sections, and assemblies are obtained as output from this stage. Then, in the *Pre-assembly* stage, the small steel components fabricated in the previous process, as webs and panels, are assembled to form the sub-blocks using welding operations. In the following stage (*Pre-outfitting*) assembled sub-blocks are internally outfitted with items like pipes, brackets, and auxiliary components. Finished sub-blocks are obtained of this stage.

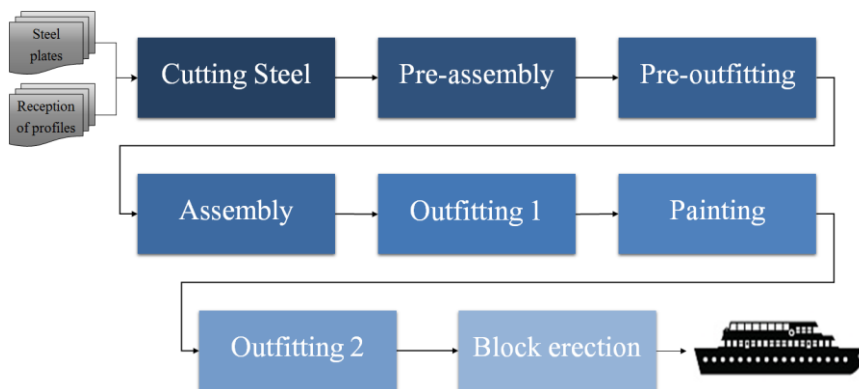


Fig. 2. Shipbuilding process

Once the blocks are outfitted, they are assembled in *Assembly* stage. The blocks assembly consists of welding operations of sub-blocks to compose a specific block. This process is carried out according to the specifications of each block. Then blocks are ready for the *Outfitting 1* process that consists of installing pipes, and electrical and lighting lines inside blocks. Part of the outfitting work is performed when the ship is upside down due to the objective is to facilitate material handling tasks. After assembling the sub-blocks to form blocks and their equipment, they are painted in the painting booths (*Painting* stage). The protection and design requirements of blocks are considered in blasting and painting operations.

A second outfitting process of blocks is performed after painting. All equipment that could be deteriorated in the painting process, such as electronic components, is installed at this *Outfitting 2* stage of the shipbuilding process.

Finally, after the painting process and the installation operations of final equipment, a *Block erection* process is carried out. Prefabricated blocks are positioned in the dry dock to build the ship, and are assembled one after another. Welding operations are also used in this stage. There is a defined order to erect these blocks, so if a block arrives earlier, it has to wait until its precedent is completed.

### 3 Mathematical model

The shipbuilding process is a complicated and long-term process that requires coordination of many different resources. Hence, a mathematical model is developed to determine the production planning for each workshop and optimize the overall shipbuilding process. In other words, the processing sequence of the blocks at each stage is optimized minimizing the total processing time.

Therefore, in this research we introduce the mathematical model based on the continuous time-slot batches concept developed for the process of shipbuilding. The qualitative problem description given in the previous section and the assumptions described below, in section 3.1, are taken into account in the proposed model. The nomenclature used in the model is detailed in section 3.2. Finally, in section 3.3 we describe all constraints that represent the features of the problem and the objective function of the model.

#### 3.1 The assumptions

The shipyard could be considered a multi-stage and multi-product plant where the parallel units in each stage are identical. Let  $I$  denote the number of blocks ( $i = 1, 2, \dots, n$ ) that the shipyard must process in the upcoming scheduling horizon. Each block is different and has its own requirements, and follows the sequence  $1, 2, \dots, c$  of stages for processing. We assume the following hypotheses for the process described above:

- There are two types of products in the shipyard: *sub-blocks* (formed by steel panels and open units) and *blocks* (formed by one or more sub-blocks).

- A unit (or workshop) cannot process more than one block (or sub-block, as appropriate) at a time. In other words, each workshop has capacity to process one block at a time.
- More than one unit cannot process a single block (or sub-block) in each stage.
- Processing units do not fail and processed blocks (or sub-blocks) are always satisfactory.
- Each block is made up of two known sub-blocks.
- The assembly sequence on slipway (the last stage of the line) is known a priori.
- The start of the current scheduling period is zero time.
- All units can start processing at time zero.
- The processing times of each block are known a priori.
- Transfer times of the blocks (or sub-blocks) between the workstations are considered negligible.
- Raw materials are unlimited.
- Intermediate storage between stages is considered NIS (non-intermediate storage).
- The production of the shipyard is programmed until the stage *Outfitting 2*, due to the output order of finished blocks of this stage is the order in which these blocks will be assembled in the last stage of shipbuilding (*Erection*).

### 3.2 Nomenclature

#### Indices.

$i, i'$	blocks
$j, j'$	sub-blocks
$s$	stages
$k$	machines or workshops
$p$	slots

#### Sets.

$I$	set of blocks (index $i, i = 1, 2, \dots, n$ )
$J$	set of sub-blocks (index $j, j = 1, 2, \dots, m$ )
$S$	set of stages (index $s, s = 1, 2, \dots, c$ )
$K$	set of machines (index $k, k = 1, 2, \dots, q$ )
$P$	set of slots (index $p, p = 1, 2, \dots, m$ )
$J_i$	set of sub-blocks of each block $i$
$K_s$	set of parallel machines in stage $s$
$I_s$	set of blocks that can be processed in stage $s$
$J_s$	set of sub-blocks that can be processed in stage $s$

#### Parameters.

$TP_{j,s}$	processing time of sub-block $i$ at stage $s$
$TPb_{i,s}$	processing time of block $i$ at stage $s$
$mc_s$	parallel units in stage $s$
$M$	big constant in big-M constraints

**Continuous variables.**

$Ti_{j,s}$	initial processing time of sub-block $j$ in stage $s$
$Tf_{j,s}$	final processing time of sub-block $j$ in stage $s$
$Tbi_{i,s}$	initial processing time of block $i$ in stage $s$
$Tfb_{i,s}$	final processing time of block $i$ in stage $s$
$TSi_{p,k}$	initial processing time of slot $p$ in machine $k$
$TSf_{p,k}$	final processing time of slot $p$ in machine $k$
$mk$	makespan

**Binary variables.**

$x_{j,p,k,s}$	1, indicates whether sub-block $j$ is processed in position $p$ of machine $k$ of stage $s$
$y_{i,p,k,s}$	1, indicates whether block $i$ is processed in position $p$ of machine $k$ of stage $s$

**3.3 Constraints**

The MILP model developed to determine the optimal production scheduling minimizing makespan includes different constraints. In the shipbuilding process we need to consider the following important constraints: the allocation constraints, the sequencing constraints, the timing constraints and the resource constraints. Following, we introduce the formulation used in the model considering these restrictions.

**Assignment of sub-blocks and blocks.**

At one time, each sub-block and block can only be processed in one workshop of each stage. The difference between equations (1) and (2) is that sub-blocks are manufactured in the first 3 stages and blocks are processed in the latter 4 stages.

$$\sum_{p=1}^N \sum_{k=1, k \in K_s}^{mc_s} x_{j,p,k,s} = 1 \quad \forall j \in J, s \in S, s \leq 3 \quad (1)$$

$$\sum_{p=1}^N \sum_{k=1, k \in K_s}^{mc_s} y_{i,p,k,s} = 1 \quad \forall i \in I, s \in S, s > 3 \quad (2)$$

**Assignment of slots.**

One sub-block (or block) can only be manufactured on one workshop, as well as one workshop can only process one sub-block. This constraint is represented in equations (3) and (4), which assigned only one sub-block (or block) in each slot of each workshop.

$$\sum_{j=1}^M x_{j,p,k,s} \leq 1 \quad \forall p \in P, k \in K_s, s \in S, s \leq 3 \quad (3)$$

$$\sum_{i=1}^N y_{i,p,k,s} \leq 1 \quad \forall p \in P, k \in K_s, s \in S, s > 3 \quad (4)$$

In addition, there should be no empty positions between consecutive sub-blocks (or blocks). The equations (5) and (6) forced to assign sub-blocks to the slots in an orderly manner.

$$\sum_{j=1}^M x_{j,(p+1),k,s} \leq \sum_{j'=1}^M x_{j',p,k,s} \quad \forall p \in P, k \in K_s, s \in S, s \leq 3, j \neq j' \quad (5)$$

$$\sum_{i=1}^N y_{i,(p+1),k,s} \leq \sum_{i'=1}^N y_{i',p,k,s} \quad \forall p \in P, k \in K_s, s \in S, s > 3, i \neq i' \quad (6)$$

#### Timing constraints.

The following equations (7)-(10) calculate final processing times of each product (sub-block and block) in each stage  $s$  of shipbuilding process, and final processing times of slots of each workshop  $k$ . Therefore, the start/end processing times which blocks and sub-blocks should fulfill to optimize a scheduling criterion such as minimum makespan are determined. Note, the binary variables  $x_{j,p,k,s}$  and  $y_{i,p,k,s}$  are used to determine the workshop and the slot in each stage products are processed.

$$Tf_{j,s} = Ti_{j,s} + \sum_p \sum_{k \in K_s} x_{j,p,k,s} * TP_{j,s} \quad \forall j \in J, s \in S, s \leq 3 \quad (7)$$

$$Tbf_{i,s} = Tbi_{i,s} + \sum_p \sum_{k \in K_s} y_{i,p,k,s} * TPb_{i,s} \quad \forall i \in I, s \in S, s > 3 \quad (8)$$

$$TSf_{p,k} = TSi_{p,k} + \sum_j x_{j,p,k,s} * TP_{j,s} \quad \forall p \in P, k \in K_s, s \in S, s \leq 3 \quad (9)$$

$$TSf_{p,k} = TSi_{p,k} + \sum_i y_{i,p,k,s} * TPb_{i,s} \quad \forall p \in P, k \in K_s, s \in S, s > 3 \quad (10)$$

#### Sequencing constraints.

The sequencing constraints restrict the processing order of the blocks and sub-blocks at each stage of the assembly process of the shipyard. Note the equations (11)-(13) differ according to the product processed at each stage  $s$ . Equation (11) correspond to stages that manufactured sub-blocks, and equation (12) represent stages that only process blocks. The assembly sequence of the sub-blocks in the *Assembly* stage ( $s = 4$ ) is modeled by equation (13).

$$Tf_{j,s} \leq Ti_{j,(s+1)} \quad \forall j \in J, s \in S, s < 3 \quad (11)$$

$$Tbf_{i,s} \leq Tbi_{i,(s+1)} \quad \forall i \in I, s \in S, s > 3 \quad (12)$$

$$Tf_{j,s} \leq Tbi_{i,(s+1)} \quad \forall i \in I, j \in J_i, s \in S, s = 3 \quad (13)$$

Equation (14) ensure that slots of the same workshop are processed in the established order:

$$TSf_{p,k} \leq TSi_{(p+1),k} \quad \forall k \in K, p \in P \quad (14)$$

The final assembly sequence required in the dry dock to meet the constraints and specification of shipbuilding process in the shipyard is modeled by the follow equation:

$$Tbf_{i,s} \leq Tbf_{i+1,s} \quad \forall i \in I, s \in S, s = |S|, i < |I| \quad (15)$$

#### Relationship between slots and blocks (or sub-blocks).

If a sub-block (or block) is processed in position  $p$  of the of machine  $k$  of stage  $s$  (i.e.  $x_{j,p,k,s} = 1$  or  $y_{i,p,k,s} = 1$ ) then the start time of the slot  $p$  must match with the start processing of the sub-block (or block). These relationships are represented by equations (16)-(19), and the constant  $M$  is used to limit the relationship between the sub-blocks (or block) and the slots. The constant  $M$  is used to limit relationships between the sub-blocks (or block) and the slots, which are represented by equations (16)-(19).

$$-M(1 - x_{j,p,k,s}) \leq Ti_{j,s} - TSi_{p,k} \quad \forall i, j \in J, p \in P, k \in K_s, s \in S, s \leq 3 \quad (16)$$

$$M(1 - x_{j,p,k,s}) \geq Ti_{j,s} - TSi_{p,k} \quad \forall i, j \in J, p \in P, k \in K_s, s \in S, s \leq 3 \quad (17)$$

$$-M(1 - y_{i,p,k,s}) \leq Tbi_{i,s} - TSi_{p,k} \quad \forall i, i \in I, p \in P, k \in K_s, s \in S, s > 3 \quad (18)$$

$$M(1 - y_{i,p,k,s}) \geq Tbi_{i,s} - TSi_{p,k} \quad \forall i, i \in I, p \in P, k \in K_s, s \in S, s > 3 \quad (19)$$

#### Objective function.

Equations (20) and (21) represent the objective function of the mathematical model. The makespan is calculated as the higher final time of the slot  $s$ , and minimized.

$$\min mk \quad (20)$$

$$mk \geq Tfb_{i,s} \quad \forall i \in I, s \in S, s > 3 \quad (21)$$

## 4 Results

The following description belongs to a small case study where a set of representative blocks and sub-blocks are proposed to show the results obtained by the continuous time-slot batches approach previously presented. In this work, real processing and assembly times are not mentioned for confidentiality reasons. Therefore, the data of



times and the configuration used of the shipbuilding system are fictitious but representative.

In the example bellow, a set of the different types of blocks  $i_1 - i_{10}$  and sub-blocks  $j_1 - j_{20}$  have to schedule in different units (or workshops). The data of processing and assembly times of each representative block (or sub-block) in each stage of shipbuilding process is presented in Table 1. Note both the transferring times and intermediate storage between consecutive stages are not considered. Table 2 shows the parallel machines in each stage of the block assembly process.

**Table 1.** Processing and assembly times of block  $i$  in each stage  $j$  (days)

<i>Block</i>	<i>Sub-block</i>	<i>Cutting Steel Time</i>	<i>Pre-assembly time</i>	<i>Pre-outfitting time</i>	<i>Assembly time</i>	<i>Outfitting 1 time</i>	<i>Painting time</i>	<i>Outfitting 2 time</i>																																																																																																																
<b>1</b>	1	60	100	15	<b>20</b>	<b>20</b>	<b>35</b>	<b>40</b>																																																																																																																
	2	60	100	15					<b>2</b>	3	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	4	50	90	25	<b>3</b>	5	60	100	15	<b>20</b>	<b>20</b>	<b>35</b>	<b>40</b>	6	60	100	15	<b>4</b>	7	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	8	50	90	25	<b>5</b>	9	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	10	50	90	25	<b>6</b>	11	60	100	15	<b>20</b>	<b>20</b>	<b>35</b>	<b>40</b>	12	60	100	15	<b>7</b>	13	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	14	50	90	25	<b>8</b>	15	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	16	50	90	25	<b>9</b>	17	60	100	15	<b>20</b>	<b>20</b>	<b>35</b>	<b>40</b>	18	60	100	15	<b>10</b>	19	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>
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	12	60	100	15					<b>7</b>	13	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	14	50	90	25	<b>8</b>	15	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	16	50	90	25	<b>9</b>	17	60	100	15	<b>20</b>	<b>20</b>	<b>35</b>	<b>40</b>	18	60	100	15	<b>10</b>	19	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	20	50	90	25																																																												
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	14	50	90	25					<b>8</b>	15	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	16	50	90	25	<b>9</b>	17	60	100	15	<b>20</b>	<b>20</b>	<b>35</b>	<b>40</b>	18	60	100	15	<b>10</b>	19	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	20	50	90	25																																																																									
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	16	50	90	25					<b>9</b>	17	60	100	15	<b>20</b>	<b>20</b>	<b>35</b>	<b>40</b>	18	60	100	15	<b>10</b>	19	50	90	25	<b>25</b>	<b>25</b>	<b>30</b>	<b>125</b>	20	50	90	25																																																																																						
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**Table 2.** Configuration of shipyard workstations

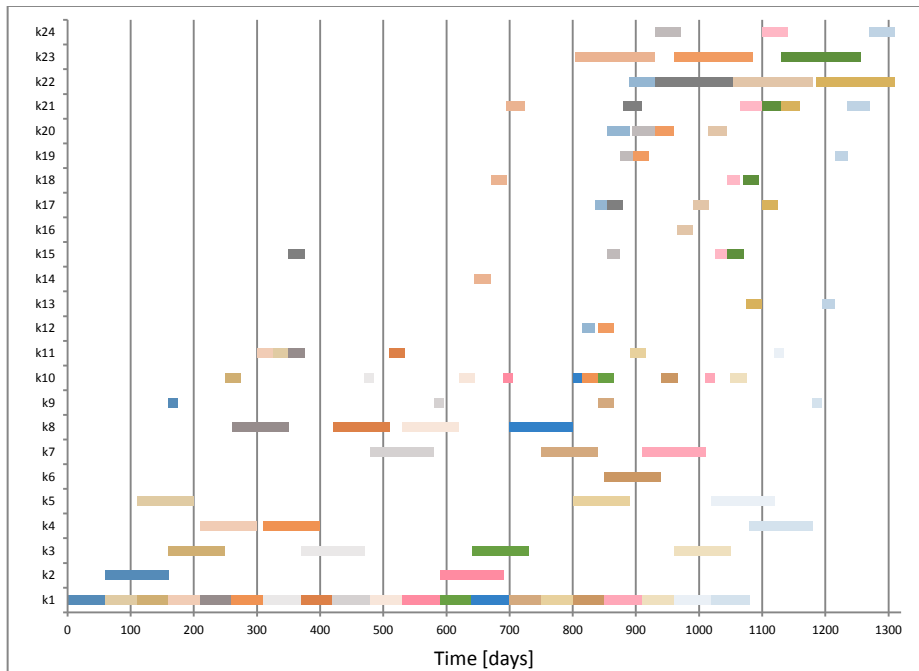
<i>Stage (s)</i>	<i>Name</i>	<i>Parallel units</i>	<i>Machines (k)</i>
1	Cutting Steel	1	$k_1$
2	Pre-assembly	7	$k_2, k_3, \dots, k_8$
3	Pre-outfitting	3	$k_9, k_{10}, k_{11}$
4	Assembly	5	$k_{12}, k_{13}, \dots, k_{16}$
5	Outfitting 1	3	$k_{17}, k_{18}, k_{19}$
6	Painting	2	$k_{20}, k_{21}$
7	Outfitting 2	3	$k_{22}, k_{23}, k_{24}$

The solver used is CPLEX with Gams software in PC Intel Core 2 Quad 2,5 GHz. The results reported in Table 3 show the main statistic of test problem analyzed for system described above (considering all blocks). The optimal solution of 1310 days is reached by the mathematical model in 5598.83 CPUs. However, the model proves a good solution with 8% relative gap in a shorter CPU time, 1791 seconds.

**Table 3.** Statistic and results of the example proposed

<i>Statistics</i>	<i>MILP model results</i>
Binary variables	7000
Continuous variables	8161
Equations	100211
Makespan (days)	1310
Gap %	0%
CPU time(s)	5598.83

Figure 3 shows the scheduling for shipbuilding process with different times and properties of the system mentioned above. The schedule of the case study proposed with 10 blocks and 20 sub-blocks is graphed. In the gantt chart that the planning horizon is 3.6 years. Notice the bottleneck of the global block assembly process can be easily identified in the schedule, .



**Fig. 3.** Solution schedule of case study proposed

New simplified systems are defined in different scenarios and then tested in the MILP model. In these scenarios, the modular construction of the ship starts with a smaller number of blocks and gradually increases up to reaching the original scenario quantity of blocks. Then, computational efficiency is determined for each one. In Table 4, the reported results show variations in model statistics when changing the number of blocks in the modular decomposition of the ship. By gradually increasing the number of blocks, a considerable increase in computational time can be observed. This remarkable growth is due to the computational size of the model (variables and equations). When the system has more than 10 blocks (and 20 sub-blocks) the model does not provide an optimal solution in an acceptable computational time.

**Table 4.** Results report for flexible number of blocks

<i>Statistics</i>	<i>Scenario 1</i>	<i>Scenario 2</i>	<i>Scenario 3</i>	<i>Scenario 4</i>	<i>Scenario 5</i>	<i>Scenario 6</i>
Blocks-Sub-blocks	4-8	5-10	6-12	7-14	8-16	9-18
Binary variables	1120	1750	2520	3430	4480	5670
Continuous variables	1585	2331	3217	4243	5409	6715
Equations	8471	15081	24491	37205	53727	74561
Makespan ( <i>mk</i> )	659	757	890	977	1090	1210
Gap %	0%	0%	0%	0%	0%	0%
CPU time(s)	3.74	14.87	35.33	179.88	935.69	169.75

## 5 Conclusions

A MILP model was developed for scheduling optimization of block assembly process of a naval industry. Results reported demonstrate that the mathematical model could obtain good-quality results in less than 1 hour of CPU time when the division in blocks of the ship does not exceed a certain number of blocks. Different scenarios were tested in order to find the best configuration, in terms of *MK* and CPU effort. Therefore, MILP-based model could be used to obtain a primary solution of real world complex scheduling problem. Future research could address the problem of bigger number of blocks combining the MILP model with other tools such as simulation and improvement algorithms.

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